

# Experiments on the Lucas Asset Pricing Model\*

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## Abstract

This paper reports on experimental tests of the Lucas asset pricing model with heterogeneous agents and time-varying private income streams. In order to emulate key features of the model (infinite horizon, stationarity, perishability of consumption), a novel experimental design was required. The experimental evidence provides broad support for the cross-sectional and inter-temporal pricing predictions of the model, but asset prices display substantial volatility unexplained by fundamentals. Consistent with Pareto efficiency under homothetic utility, consumption shares of the two types of agents in our experiment are constant across states and time; under autarky, consumptions would have been *negatively* correlated. Generalized Method of Moments (GMM) tests reject the asset pricing restrictions. The paper suggests that the coexistence of bad prices (excess volatility) and good allocations (Pareto efficiency) arises from participants' expectations about future prices, which are at odds with the theoretical predictions of the Lucas model but are nonetheless *almost* self-fulfilling.

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# 1 Introduction

For over thirty years, the Lucas asset pricing model (Lucas, 1978) has served as the basic platform for research on dynamic asset pricing and business cycles. The Lucas model provides both cross-sectional and time-series predictions and links the two. The central cross-sectional prediction is parallel with the central predictions of static models such as CAPM (the Capital Asset Pricing Model): only aggregate risk is priced. In CAPM aggregate risk is measured by the return on the market portfolio, and the price of an asset decreases (the return on the asset increases) with the “beta” of the asset (the covariance of the return on the asset with the return on the market portfolio). In the Lucas model, aggregate risk is measured by aggregate consumption, and the price of an asset decreases (the return increases), with the “consumption beta” of the asset. The central time-series predictions of the Lucas model are that asset price changes are correlated with economic fundamentals (aggregate consumption growth) and that there is a strong connection between the volatility of asset prices and the volatility of economic fundamentals. The most important consequence of this prediction is that asset prices need *not* follow a martingale (with respect to the true probabilities) and the price of an asset need *not* be the discounted present value of its expected future dividends (with respect to the true probabilities). These contradict the strictest interpretation of the Efficient Markets Hypothesis (Samuelson, 1973; Malkiel, 1999; Fama, 1991).<sup>1</sup>

The most familiar version of the Lucas model assumes a representative agent, whose holdings consist of the aggregate endowment of securities and whose consumption is the aggregate flow of the (perishable) dividends. Asset prices are constructed as shadow prices with respect to which the representative agent would have no incentive to trade. The representative agent has rational expectations, and so correctly forecasts both future prices and his own future decisions. The multi-agent version of the Lucas model that we study here assumes that all agents have rational expectations, and so correctly forecast both future prices and their own future decisions, and that prices and allocations form an equilibrium; in particular, allocations are Pareto optimal and agents (optimally) smooth consumption over time and states of nature. Although the quantitative predictions of the representative agent model and the heterogeneous agent model may differ, the qualitative predictions are the same.

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<sup>1</sup>Because prices do not admit arbitrage, the Fundamental Theorem of Asset Pricing implies the existence of *some* probability measure typically different from the true probability measure with respect to which prices *do* follow a martingale – but that is a tautology, not a prediction.

This paper reports on experimental laboratory tests of the Lucas model with heterogeneous agents. We find experimental evidence that provides broad support for the cross-sectional and intertemporal pricing predictions and for the consumption smoothing/risk sharing predictions of the theory – but with significant and notable differences. On the one hand, as theory predicts, asset prices co-move with economic fundamentals and this co-movement is stronger when cross-sectional price differences are greater. On the other hand, asset prices are significantly more volatile than fundamentals account for (fundamentals explain only a small fraction of the variance of price changes) and returns are less predictable than theory suggests. (Indeed, for the (consol) bond, the noise in the price data is so great that we cannot reject the null that price changes are entirely random, unrelated to fundamentals.) The data suggest that the divergence from theoretical predictions arises from subjects’ forecasts about future asset prices, which appear to be vastly at odds with the predictions of the Lucas model, yet *almost self-fulfilling*. Of course asset price forecasts that are *exactly* self-fulfilling must necessarily coincide with the prices predicted by the Lucas model — this is just the definition of equilibrium in the model. Surprisingly, however, asset price forecasts can be almost self-fulfilling and yet far from the equilibrium prices and in particular far from the predictions of the Lucas model. Among other things, these findings suggest that excessive volatility of prices may not be indicative of large welfare losses.

Up to now, analysis of the Lucas model, both empirical and theoretical, has traditionally focused on the “stochastic Euler equations” that deliver the equilibrium pricing restrictions (Cochrane, 2001). These equations derive from the first-order conditions of the consumption/investment optimization problem of the representative agent in the economy. Empirical tests of the stochastic Euler equations on historical field data have been disappointing; indeed, beginning with Mehra and Prescott (1985), the fit of model to data has generally been considered to be poor. Attempts to improve the fit of the model to data have concentrated on the auxiliary assumptions rather than on its primitives. Some authors have altered the original preference specification (time-separable expected utility) to allow for, among others, time-nonseparable utility (Epstein and Zin, 1991), loss aversion (Barberis et al., 2001), or utility functions that assign an explicit role to an important component of human behavior, namely, emotions (such as disappointment; Routledge and Zin (2011)). Others have looked at measurement problems, extending the scope of aggregate consumption series in the early empirical analysis (Hansen and Singleton, 1983); the role of durable goods (Dunn and Singleton, 1986); the role of certain goods as providing collateral as well as consumption (Lustig and Nieuwerburgh, 2005); and the presence of a small-amplitude, low-frequency com-

ponent in consumption growth along with predictability in its volatility (Bansal and Yaron, 2004).

By contrast, our experimental study of the Lucas model focuses on the primitives of the model, rather than merely trying to find an instantiation of the stochastic Euler equations that best fits a given series of price (and aggregate consumption) data. In the laboratory, we will be able to examine all predictions of the model – not just whether prices satisfy some set of stochastic Euler equations. This is possible because the laboratory environment allows us to observe structural information that is impossible to glean from historical data, such as aggregate supplies of securities, beliefs about dividend processes, and private income flows.<sup>2</sup> In the laboratory, we are able to observe all the important variables and control many of them (with the notable exception of participants’ preferences).

However, the nature of the Lucas model presents a number of unusual challenges for the laboratory environment. Most obviously, the classic version assumes a representative agent, or equivalently a collection of identical agents – which would seem unlikely in any realistic setting and is certainly an absurdity in a laboratory environment, where heterogeneity is almost guaranteed, at least with respect to preferences. (We introduce heterogeneity of endowments as well in order to stimulate trade, which helps agents to learn the price process.) As we shall see, the predictions of the heterogeneous agent model are qualitatively no different than the predictions of the representative agent model, but they arise in a different way. In the representative agent model, Pareto optimality is tautological – there is after all, only one agent. In the heterogeneous agent model, Pareto optimality can arise only if agents can trade and is not guaranteed even then; it is only guaranteed if trade leads to a Walrasian equilibrium.

Walrasian equilibrium would seem to require complete markets, and our laboratory markets are far from complete – indeed only two assets, a Bond and a Tree are traded. However, our laboratory markets are, if not complete, at least (potentially) *dynamically complete*. That is, in a Radner equilibrium (the appropriate notion for an economy such as the one we create), the effect of complete markets can be replicated by frequent trading of the long-lived assets (Duffie and Huang, 1985). However, for dynamic completeness to emerge, participants must employ complex investment policies that exhibit the *hedging* features that are at the core of the modern theory of derivatives analysis (Black and Scholes, 1973; Merton, 1973a) and dynamic asset pricing (Merton, 1973b). Moreover, investors would need to make correct forecasts of future (equilibrium) prices – because that is what a Radner perfect foresight equilib-

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<sup>2</sup>Note the similarity to the Roll (1977) critique.

rium (Radner, 1972) requires. For tractability in the laboratory we treat a model with only two securities: a (consol) Bond whose dividend each period is fixed and a Tree whose dividend follows an announced known stochastic process. In contrast to the literature on “learning rational expectations equilibrium” agents in our experimental economy do not need to learn/forecast the exogenous uncertainty – it is told to them. However they still must learn/forecast the *endogenous* uncertainty – the uncertainty about future prices.

In addition to the heterogeneity of agents, three particularly challenging aspects of the Lucas model need to be addressed before one can test it in the laboratory. The model assumes that the time horizon is infinite, that the environment is stationary, and that investment demands are driven primarily by the desire to smooth consumption. We deal with the infinite horizon as in Camerer and Weigelt (1996), by introducing a random ending time. As is well-known, a stochastic ending time is (theoretically) equivalent to discounting over an infinite time horizon (assuming subjects are expected utility maximizers with time-separable preferences). However the laboratory imposes some additional complications. Because the experiment necessarily lasts for a limited amount of time, the beliefs of participants about the termination probability are likely to change when the duration of the session approaches the officially (or perceived) announced limit. If subjects believe the termination probability is non-constant, a random ending time would correspond to a non-constant discount factor; worse yet, different subjects might have different beliefs and hence different discount factors. For the same reason, there would be an issue about stationarity. To treat this problem we introduce a novel treatment: we adopt a termination rule that is (theoretically) equivalent to an infinite horizon with constant discounting or constant termination probability. Finally, because it is hard to imagine that participants would care about the timing of their consumption (earnings) across periods during the course of an experiment, we introduce another novel treatment: we emulate perishability by imposing forfeiture of participants’ cash holdings (the consumption good) at the end of every non-terminal period: cash held at the end of the randomly determined terminal period – and only then – is “consumed” (taken home as experimental earnings). As we show, optimization in this environment is equivalent to maximizing discounted lifetime expected utility. The desire to smooth consumption is a consequence of this perishability and the risk aversion that subjects bring to the laboratory.

In parallel work, Crockett and Duffy (2010) also study an infinite horizon asset market in the laboratory, but their experimental approach and purpose are very different from ours. In particular, their approach to consumption smoothing is to *induce* a

preference for consumption smoothing imposing a schedule of final payments to participants that is non-linear in period earnings. A problem with that approach – aside from the question of whether one should try to induce preferences rather than take them as given – is that this is (theoretically) equivalent to time-separable additive utility *only* if participant’s true preferences are risk-neutral – but there is ample laboratory evidence that participants display substantial risk-aversion even for relatively small laboratory stakes Bossaerts and Zame (2008). Moreover, because their focus is different from ours – their focus is on bubbles, ours is on the primitive implications of the model – they create an environment and choose parameters that are conducive to *little* trading, while we create an environment and choose parameters that are conducive to *much* trading.

The remainder of this paper is organized as follows. Section 2 presents the Lucas model within the framework of the laboratory economy we created. Section 3 provides details of the experimental setup. Results are provided in Section 4. Section 5 discusses potential causes behind the excessive volatility of asset prices observed in the laboratory markets. Section 6 examines the laboratory data through the lens of the statistical analysis that has traditionally been employed on historical field data. Section 7 concludes.

## 2 The Lucas Asset Pricing Model

We use (a particular instantiation of) the Lucas asset pricing model with heterogeneous agents that is simple enough to implement in the laboratory and yet complex enough to generate a rich set of predictions about prices and allocations. As we shall see, testable predictions emerge under very weak assumptions (allowing complete heterogeneity of endowments and preferences across agents); stronger predictions emerge under stronger assumptions (identical preferences). Because we wish to take the model to the laboratory setting, a crucial feature of our design is that it generates a great deal of trade; indeed Pareto optimality (hence equilibrium) requires that trading takes place *every period*. This is important in the laboratory setting because subjects do not know the “correct” equilibrium prices (nor do we) and can only learn them through trade, which would seem problematic (to say the least) if theory predicted that trade would take place infrequently. We therefore follow Bossaerts and Zame (2006) and insist that individual endowments not be stationary (where by “stationary” we mean “to be a time-invariant function of dividends”) – although aggregate endowments are

stationary, which is a key assumption of the Lucas model.<sup>3</sup> As Crockett and Duffy (2010) confirm, not giving subjects a reason to trade in every period – or at least frequently – is a recipe for producing price bubbles in the laboratory, perhaps because subjects are motivated to trade out of boredom rather than for financial gain.)

We caution the reader that we use the original Lucas model, which assumes stationarity in *dividend levels* and not in *dividend growth*. Beginning with Mehra and Prescott (1985), the models that have used historical field data to inform empirical research assume stationarity in *growth rates*. We choose stationarity in levels because it is easier to implement in the laboratory – an important (perhaps necessary) condition for an experiment that already poses many other challenges. While the main message of the two versions of the Lucas model is much the same – e.g., prices move with fundamentals – there are also important qualitative (and quantitative) differences.

## 2.1 A General Environment

We consider an infinite horizon economy with a single consumption good in each time period (in the experiment, the consumption good is cash so we use ‘consumption’ and ‘cash’ interchangeably here). In each period there are two possible states of nature  $H$  (high),  $L$  (low), which occur with probabilities  $\pi, 1 - \pi$  independently of time and past history. Two long-lived assets are available for trade: (i) a (*consol*) *Bond* that pays a constant dividend  $d_B$  each period, and (ii) a *Tree* that pays a stochastic dividend  $d^H$  when the state is  $H$ ,  $d^L$  when the state is  $L$ ;. We assume  $d_T^H > d_T^L \geq 0$  and normalize so that  $d_B = \pi d_T^H + (1 - \pi) d_T^L$ ; i.e., the Bond and the Tree have the same expected dividend. Note that the dividends processes are stationary. With little loss of generality, and in line with the experiment, we assume that  $\pi = 1/2$ , and  $d_T^H = 1$ ,  $d_T^L = 0$ , so that  $d_B = 0.5$ .

There are  $n$  agents. Each agent  $i$  has an initial endowment  $b_i$  of bonds and  $\tau_i$  of trees, and also receives an additional endowment of consumption  $e_{i,t}$  (possibly random) in each period  $t$ . Write  $b = \sum b_i$ ,  $\tau = \sum \tau_i$  and  $e = \sum e_i$  for the social (aggregate) endowment of bonds, trees and additional consumption in the form of private income flows. We assume that the social endowment of  $e$  is stationary (meaning that it is a time-invariant function of dividends – in the experiment, it will be constant) but we impose *no* restriction on individual endowments. (As noted earlier, we wish to ensure that in the experimental setting subjects have a reason to trade each period.)

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<sup>3</sup>As Judd et al. (2003) has shown, if individual endowments, as well as aggregate endowments, are stationary then at equilibrium all trading takes place in the initial period.

Each agent  $i$  maximizes expected lifetime utility for infinite (stochastic) consumption streams

$$U_i(\{c_t\}) = E \left[ \sum_{t=1}^{\infty} \beta^{t-1} u_i(c_t) \right]$$

where  $c_t$  is (stochastic) consumption at time  $t$ . We assume that the period utility functions  $u_i$  are smooth, strictly increasing, strictly concave and have infinite derivative at 0 (so that optimal consumption choices are interior). Note that agent endowments and utility functions are heterogeneous but that all agents use the same constant discount factor  $\beta$ . (In the experimental setting this seems an especially reasonable assumption because the discount factor is just the probability of continuation, which is constant and common across agents.)

In each period  $t$  agents receive dividends from the Bonds and Trees they hold, trade their holdings at current prices, use the proceeds together with their endowments to buy a new portfolio of Bonds and Trees, and consume the remaining cash. Agents take as given the current prices of the bond  $p_{B,t}, p_{T,t}$  (which depend on the current state) but must make forecasts of (stochastic) future asset prices  $p_{B,t'}, p_{T,t'}$  for each  $t' > t$  and optimize subject to their current budget constraint and the forecast future path of prices. (Implicitly, agents optimize subject to their forecast future path of consumption choices). At a *Radner equilibrium* (Radner, 1972) markets for consumption and assets clear at every date and state and all price forecasts are correct. This is not quite enough for equilibrium to be well-defined because it does not rule out the possibility that agents acquire more and more debt, pushing debt further and further into the future and never repaying it. Levine and Zame (1996), Magill and Quinzii (1994) and Hernandez and Santos (1996) show that it is sufficient to add a requirement that bounds debt. Levine and Zame (1996) show that all ‘reasonable’ choices lead to the same equilibria; the simplest is to require that debt not become unbounded. (Lucas (1978) finesses the problem in a different way by defining equilibrium to consist of prices, choices and a value function – but if unbounded debt is permitted then no value function can possibly exist.)

As is universal in the literature we assume that a Radner equilibrium exists and – because markets are (potentially) dynamically complete – that it coincides with Walrasian equilibrium and in particular that equilibrium allocations are Pareto optimal.<sup>4</sup>

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<sup>4</sup>These assumptions may disturb the reader. But, as pointed out before, the familiar version of the Lucas model *starts* by assuming that allocations are Pareto optimal, and exploits the resulting existence of a representative agent to derive prices. As such, all that we are assuming is *subsumed* in the familiar version. Unless of course one views the familiar Lucas model as the outcome of a world where every agent is identical



## 2.2 Predictions

Despite the absence of assumptions about the functional form of utility functions, the model above does make *quantitative* predictions. Our assuming only two possible states each period (High or Low dividend on the Tree) allows us to translate the usual *qualitative* predictions into statements that can be quantified – up to a certain extent. Most of these predictions are entirely familiar in the context of the usual Lucas model which assumes a representative agent with CRRA utility; we offer them at this point to emphasize that they do not rest on the assumption of a representative agent or any particular parameters or functional forms. (Of course we make no claim that any of these observations is original.) In the next subsection, we will provide explicit numerical solutions when everyone displays logarithmic utility.

### 1. Individual consumption is stationary and perfectly rank-correlated.

To see this, fix a period  $t$ . The boundary condition guarantees that equilibrium allocations are interior, so smoothness and Pareto optimality guarantee that all agents have the same marginal rate of substitution for consumption in state  $H$  at periods  $t, t + 1$ . Market clearing implies that social consumption equals the aggregate amount of dividends and individual consumption endowments. The latter is stationary, hence equal in state  $H$  at periods  $t, t + 1$ . It follows that the consumption of each individual agent must also be equal in state  $H$  at periods  $t, t + 1$ ; since  $t$  is arbitrary this means that individual consumption must be constant in state  $H$ . Similarly, individual consumption must be constant in state  $L$ . It also follows that, across states, all agents rank marginal utilities of consumption in the same order. Strict concavity of period utility functions implies that all agent rank levels of consumption in the same order as well (but opposite to marginal utilities). Consequently, equilibrium individual consumptions are stationary and perfectly rank-correlated across states.

### 2. The Euler equations obtain.

To see this, fix an agent  $i$ ; write  $\{c_i\}$  for  $i$ 's stochastic equilibrium consumption stream (which we have just shown to be stationary). Because  $i$  optimizes given current and future asset prices, asset prices in period  $t$  must equalize marginal utility of consumption at each state in period  $t$  with expected marginal utility of consumption at period  $t + 1$ . If  $i$  buys (sells) an additional infinitesimal amount  $\varepsilon$  of an asset at period  $t$ , consumption in period  $t$  is reduced (increased) by  $\varepsilon$  times

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(at which point the representative agent exists trivially). This world is neither the one we encounter in the field nor in our experiments.

the price of the asset but consumption in period  $t + 1$  is increased (reduced) by  $\varepsilon$  times the delivery of the asset, which is the sum of its dividend and its price in period  $t + 1$ . Hence the first order condition is:

$$p_{B,t}^H = \beta \left\{ \pi \left[ \frac{u'_i(c_i^H)}{u'_i(c_i^H)} \right] (d + p_{B,t+1}^H) + (1 - \pi) \left[ \frac{u'_i(c_i^L)}{u'_i(c_i^H)} \right] (d + p_{B,t+1}^L) \right\}$$

where superscripts index states and subscripts index assets, time, agents in the obvious way. The obviously analogous identities hold for the state  $L$  and for the tree, so we can write these equations in more compact form as

$$p_{k,t}^s = \beta E \left\{ \left[ \frac{u'_i(c_i)}{u'_i(c_i^s)} \right] (d_k + p_{k,t+1}) \right\} \quad (1)$$

for  $s \in \{H, L\}$  and  $k \in \{B, T\}$ . (1) is the familiar Euler equation, except that the marginal utilities are that of an arbitrary agent  $i$  and not of the representative agent. Equality of the ratios of marginal utilities across agents, which is a consequence of Pareto optimality, of course implies that we could write (1) in terms of the utility function of a representative agent, but notice that that this utility function is determined *in equilibrium*.

We can let  $x$  denote the ratio of marginal utilities of the state transition from  $H$  (the tree pays a dividend of \$1) to  $L$  (the tree pays no dividend) (i.e., the marginal rate of substitution of consumption in  $L$  and  $H$ ). Because of risk aversion,  $x > 1$ . We can then solve equation (1), to obtain:

$$p_{B,t}^H = \frac{\beta}{1 - \beta} \frac{x + 1}{2} 0.5 \quad (2)$$

$$p_{B,t}^L = \frac{\beta}{1 - \beta} \frac{x + 1}{2x} 0.5 \quad (3)$$

$$p_{T,t}^H = \frac{\beta}{1 - \beta} 0.5 \quad (4)$$

$$p_{T,t}^L = \frac{\beta}{1 - \beta} \frac{1}{x} 0.5 \quad (5)$$

From (4), it follows that the price of the tree in state  $H$  is independent of risk attitudes (as embedded in  $x$ ), and solely dependent on impatience ( $\beta$ ). If  $\beta$  equals  $5/6$ , for instance,  $p_{T,t}^H = 2.5$  always.

### 3. Asset prices are stationary.

This follows immediately from equations (2) to (5).

### 4. Asset prices are correlated with fundamentals.

This is also an immediate consequence of equations (2) to (5). Informally, this

is understood most clearly by thinking about the representative agent. In state  $H$ , aggregate consumption supply is high, so high prices (low returns) must be in place to abate the representative agent's desire to save (invest). The opposite is true for state  $L$  – aggregate consumption is low, so the representative agent would wish to borrow (sell) if it weren't for the low prices (high returns).

### 5. The Tree is cheaper than the Bond.

This too is a consequence of equations (2) to (5). In the context of static asset-pricing theory this pricing relation is a simple consequence of the fact that the dividends on the Tree have higher covariance with aggregate consumption than does the Bond; in other words, the Tree has higher “beta” than the Bond. However, in the dynamic context the result is more subtle because asset prices in period  $t$  depend on dividends in period  $t + 1$  *and* on asset prices in period  $t + 1$ ; since prices are determined in equilibrium, it does not seem clear *a priori* that *prices* of the Tree have higher covariance with aggregate consumption than prices of the Bond.

The ratio of prices in the High and Low states is constant across assets:

$$\frac{p_{B,t}^H}{p_{B,t}^L} = \frac{p_{T,t}^H}{p_{T,t}^L} = x \ (> 1).$$

The difference in the prices of the Tree and the Bond can be translated into differences in expected returns. The difference between the expected return on the security representing the risk in the economy (the Tree) and that of a (relatively) risk free security (the Bond) is known as the *equity premium* (Mehra and Prescott, 1985). The conclusion that the Tree is cheaper than the Bond implies that the equity premium is positive.

Specifically, tedious computations show that the equity premium in the  $H$  state,  $E_t^H$ , equals:

$$E_t^H = \frac{\beta}{1 - \beta} \frac{2x + 1}{2(x + 1)},$$

while in the  $L$  state,  $E_t^L$  equals:

$$E_t^L = \frac{\beta}{1 - \beta} (x - 1).$$

Both expressions are positive.

### 6. The equity premium is counter-cyclical.

This follows immediately from the above equations. Specifically,

$$E_t^H - E_t^L = \frac{\beta}{1 - \beta} \frac{-2x^2 + 2x - 1}{2(x + 1)},$$

which is strictly negative for values of  $x$  above 1. When the equity premium is lower in the High than the Low state, it is said to be counter-cyclical. The counter-cyclicity provides the correct incentives: when dividends are low, the equity premium is high, so investors buy risky Trees rather than consuming scarce dividends; when dividends are high, the equity premium is low, so investors prefer to consume rather than engage in risky investment.

Conversely, the discount of the price of the Tree relative to that of the Bond ( $p_B^s - p_T^s$ ,  $s = H, L$ ) is pro-cyclical. This follows directly from the fact that the ratio of the prices across states of both securities are equal and the fact that the Bond is always more expensive than the Tree.

### 7. Asset prices and returns are predictable.

Asset prices are predictable because they depend on the state; see equations (2) to (5). Returns are predictable, which is a just a simple re-formulation of the prediction that the equity premium is counter-cyclical. Predictability of prices (and returns) obtains in stark contrast with simple versions of the Efficient Markets Hypothesis (EMH), which states that prices are a martingale under the true probabilities (Samuelson, 1973; Malkiel, 1999; Fama, 1991).

### 8. Cross-sectional and time series properties of asset prices reinforce each other.

To be more precise, as the discount of the Tree price relative to the Bond price increases because risk aversion rises, the difference in Tree prices or in Bond prices across states increases. That is,

$$\text{cov}(p_{B,t}^s - p_{T,t}^s, p_{k,t}^H - p_{k,t}^L) > 0,$$

for  $s = H, L$  and  $k = B, T$ , and where the covariance is computed based on sampling across cohorts of agents (economies), keeping everything else constant. “Everything else” means: initial endowments, private income flows, asset structure, outcome probabilities, as well as impatience  $\beta$ . Economies are therefore distinguishable at the price level only in terms of the risk aversion (embedded in  $x$ ) of the representative agent.<sup>5</sup>

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<sup>5</sup>To obtain the result, write all variables in terms of  $x$ :

$$\begin{aligned} p_{B,t}^H - p_{T,t}^H &= 0.5^2 \frac{\beta}{1-\beta} (x-1) \\ p_{B,t}^L - p_{T,t}^L &= -0.5^2 \frac{\beta}{1-\beta} \frac{1}{x} + \text{constant} \end{aligned}$$

### 9. Agents smooth consumption.

Individual equilibrium consumptions are stationary but individual endowments are not, so agents smooth over time.

### 10. Agents trade to hedge price risk.

If there were no price risk, agents could smooth consumption simply by buying or selling one asset. However, there is price risk, because prices move with fundamentals and fundamentals are risky. Hence, when agents sell assets to cover a private income shortfall (where shortfall is in relation to the aggregate average private income), they also need to insure against the risk that prices might change by the time they are ready to buy back the assets. In equilibrium, prices increase with the dividend on the Tree, and agents correctly anticipate this. Since the Tree pays a dividend when prices are high, it is the perfect asset to hedge price risk. Consequently (but maybe counter-intuitively!), agents *buy* Trees in periods with income shortfall and they *sell* when their income is high.

Hedging is usually associated with Merton's intertemporal asset pricing model (Merton, 1973b) and is the core of modern derivatives analysis (Black and Scholes, 1973; Merton, 1973a). Here, it forms an integral part of the trading predictions of the Lucas model.

It can be shown that price risk hedging increases with the risk aversion of the representative agent. This is because equilibrium price risk, measured as the difference in prices across  $H$  and  $L$  states, increases with risk aversion (embedded in  $x$ ).

In summary, our implementation of the Lucas model predicts that securities prices differ cross-sectionally depending on consumption betas (the Tree has the higher beta), while intertemporally, securities prices move with fundamentals (dividends of the Tree). The two predictions reinforce each other: the bigger the difference in prices across securities, the larger the intertemporal movements. Investment choices should be such that consumption (cash holdings at the end of a period) across states becomes perfectly rank-correlated between agent types (or even perfectly correlated, if agents have the same preferences). Likewise, consumption should be smoothed across periods with and

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$$\begin{aligned} p_{B,t}^H - p_{B,t}^L &= \frac{\beta}{1-\beta} \frac{x}{4} + \text{constant} \\ p_{T,t}^H - p_{T,t}^L &= -0.5 \frac{\beta}{1-\beta} \frac{1}{x} + \text{constant} \end{aligned}$$

All variables increase in  $x$  (for  $x > 1$ ). As  $x$  changes from one agent cohort (economy) to another, these variables all change in the same direction. Hence, across agent cohorts, they are positively correlated.

Table 1: Prices, discounts on the Tree relative to the Bond, and equity premiums, as functions of the state (High  $H$ /Low  $L$ )

State	Tree		Bond		Price	Equity
	Price	Return	Price	Return	Discount	Premium
High ( $H$ )	\$2.50	3.4%	\$3.12	-0.5%	\$0.62	3.9%
Low ( $L$ )	\$1.67	55%	\$2.09	49%	\$0.42	6%

without income. Investment choices are sophisticated: they require, among others, that agents hedge price risk, by buying Trees when experiencing income shortfalls (and selling Bonds to cover the shortfalls), and selling Trees in periods of high income (while buying back Bonds).

### 2.3 Numerical Example

Here, we compute equilibrium prices, holdings and consumption assuming that agents display logarithmic utility. In addition, we take the structure of endowments as in the experiment.

- There are an even number  $n = 2m$  of agents; agents  $i = 1, \dots, m$  are of Type I, agents  $i = m + 1, \dots, 2m$  are of Type II.
- Type I agents are endowed with asset holdings  $b_I = 0, \tau_I = 10$  and have income  $e_{I,t} = 15$  when  $t$  is even and  $e_{I,t} = 0$  when  $t$  is odd.
- Type II agents are endowed with asset holdings  $b_{II} = 10, \tau_{II} = 0$  and have income  $e_{II,t} = 15$  when  $t$  is odd and  $e_{II,t} = 0$  when  $t$  is even.

Table 1 provides equilibrium asset prices, the discounts in the price of the Tree relative to the Bond, and equity premia, as functions of the state and of risk aversion. As expected, Trees are always cheaper than Bonds. The discount on the Tree is higher in state  $H$  than in state  $L$ , while the equity premium is lower in state  $H$  than in state  $L$ , reflecting the pro-cyclical behavior of the discount and the counter-cyclical behavior of the equity premium. The dependence of prices on the state, and the predictability of returns is apparent from the table.<sup>6</sup>

<sup>6</sup>From Equation 1, one can derive the (shadow) price of a one-period pure discount bond with principal of \$1, and from this price, the one-period risk free rate. In the High state, the rate equals -4%, while in the low state, it equals 44%. As such, the risk free rate mirrors changes in expected returns on the Tree and Bond. The reader can easily verify that, when defined as the difference between the expected return on

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Table 2: Type I agent equilibrium holdings and trades as a function of period (Odd/Even); Type I agents receive income in Even periods only.

Period	Tree	Bond	(Total)
Odd	7.57	0.62	(8.19)
Even	2.03	7.78	(9.81)
(Trade in Odd)	(+5.54)	(-7.16)	(-1.62)

---

Table 2 provides equilibrium holdings and trades for Type I agents (who receive income in Even periods, and hence, need to overcome consumption shortfall in Odd periods). As expected, the lack of income in Odd periods is resolved *not* through outright sales of assets, *but* through a combination of sales of Bonds and purchases of Trees. The Bond sales provide income; the Tree purchases ensure that the risk of price changes between Odd periods (when Type I agents are net sellers of assets) and Even periods (when Type II agents are net buyers of assets) is hedged.<sup>7</sup>

Equilibrium holdings and trades ensure that Type I agents (and consequently, Type II agents as well) consume a *constant fraction* of total available consumption in the economy, namely 48%. This consumption share is independent of state (High/Low) or period (Odd/Even). Constancy of consumption shares obtains if the allocations are Pareto optimal and agent utilities are homothetic. Constant consumption sharing is a stronger result than the perfect rank correlation one obtains in general (see the first prediction in the previous subsection) because it implies that consumption will be perfectly correlated across agents.

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the market portfolio (the per-capita average portfolio of Trees and Bonds) and the risk free rate, the equity premium is countercyclical, just like it is when defined as the difference between the expected return on the Tree and on the Bond.

<sup>7</sup>Equilibrium holdings and trade do *not* depend on the state (dividend of the Tree). However, they do depend on the state in Period 1. Here, we assume that the state in Period 1 is *H* (i.e., the Tree pays a dividend of \$1). If the state in Period 1 were *L*, there would be a technical problem when risk aversion is greater than 0.5: in Odd periods, agents would need to short sell Bonds. Short sales were not allowed in the experiment.

### 3 Implementing the Lucas Model

As we have already noted, implementing the Lucas economy in the laboratory encounters three difficulties:

- (a) The Lucas economy has an infinite horizon, but an experimental session has to end in finite time.
- (b) There is no natural demand for consumption smoothing in the laboratory. Because actual consumption is not feasible until after an experimental session concludes, it would not make much of a difference if we were to pay subjects' earnings gradually, over several periods.
- (c) The Lucas economy is stationary.

In our experiment, we used the standard solution to resolve issue (a), which is to randomly determine if a period is terminal (Camerer and Weigelt, 1996). This ending procedure also introduces discounting: the discount factor will be proportional to the probability of continuing the session. We set the termination probability equal to  $1/6$  so the continuation probability, which is the induced discount factor, is  $\beta = 5/6$ . In mechanical terms: after the markets in period  $t$  closed we rolled a twelve-sided die; if it came up either 7 or 8, we terminated; otherwise we moved on to a new period.

To resolve issue (b), we made end-of-period individual cash holdings *disappear* in every period that was not terminal; only securities holdings carried over to the next period. If a period *was* terminal, however, securities holdings perished and cash holdings were credited; participants' earnings were then determined entirely by the cash they held at the end of this terminal period. As such, if participants have expected utility preferences, their preferences will automatically become of the time-separable type that Lucas used in his model, albeit with an adjusted discount factor: the period- $t$  discount factor becomes  $(1 - \beta)\beta^{t-1}$ , so utility is multiplied by  $(1 - \beta)$ .<sup>8</sup> Of course, multiplying utility by a positive constant has no effect on choices or prices.

It is less obvious how to resolve problem (c). In principle, the constant termination probability would do the trick: any period is equally likely to be terminal. This does

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<sup>8</sup>Starting with Epstein and Zin (1991), it has become standard in research on the Lucas model with historical field data to use time-nonseparable preferences, in order to allow risk aversion and intertemporal consumption smoothing to affect pricing differentially. Because of our experimental design, we cannot appeal to time-nonseparable preferences if we need to explain pricing anomalies. Indeed, separability across time and states is a natural consequence of expected utility. We consider this to be a strength of our experiment: we have tighter control over preferences. This is addition to our control of beliefs: we make sure that subjects understand how dividends are generated, and how termination is determined.



imply, however, that the chance of termination does not depend on how long the experiment has been going, and therefore, the experiment could go on forever, or at least, take much longer than a typical experimental session. Our own pilots confirmed that subjects' beliefs were very much affected as the session reached the 3 hour limit. We employed a simple solution, exploiting essential features of the Lucas model. We announced that the experimental session would last until a pre-specified time and there would be as many replications of the (Lucas) economy as could be fit within this time frame. If a replication finished at least 10 minutes before the announced end time, a new replication started; otherwise, the experimental session was over. *If a replication was still running by the closing time, we announced before trade started that the current period was either the last one (if our die turned up 7 or 8) or the next-to-last one (for all other values of the die).* In the latter case, we moved to the next period and this one became the terminal one with certainty. This meant that subjects would keep the cash they received through dividends and income for that period. (There will be no trade because assets perish at the end, but we always checked to see whether subjects correctly understood the situation.) In the Appendix, we re-produce the time line plot that we used alongside the instructions to facilitate comprehension.

It is straightforward to show that the equilibrium prices remain the same whether the new termination protocol is applied or if termination is perpetually determined with the roll of a die. In the former case, the pricing formula is:<sup>9</sup>

$$p_{k,t} = \frac{\beta}{1-\beta} E\left[\frac{u'_i(c_{i,t+1})}{u'_i(c_{i,t})} d_{k,t+1}\right]. \quad (6)$$

To see that the above is the same as the formula in Eqn. (1), apply the assumption of i.i.d. dividends and the consequent stationary investment rules (which generate i.i.d. consumption flows) to re-write Eqn. (1) as an infinite series that can easily be solved:

$$p_{k,t} = \sum_{\tau=0}^{\infty} \beta^{\tau+1} E\left[\frac{u'_i(c_{i,t+\tau+1})}{u'_i(c_{i,t+\tau})} d_{k,t+\tau+1}\right]$$

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<sup>9</sup>To derive the formula, consider agent  $i$ 's optimization problem in period  $t$ , which is terminal with probability  $1-\beta$ , and penultimate with probability  $\beta$ , namely:  $\max (1-\beta)u_i(c_{i,t}) + \beta E[u_i(c_{i,t+1})]$ , subject to a standard budget constraint. The first-order conditions are, for asset  $k$ :

$$(1-\beta) \frac{\partial u_i(c_{i,t})}{\partial c} p_{k,t} = \beta E\left[\frac{\partial u_i(c_{i,t+1})}{\partial c} d_{k,t+1}\right].$$

The left-hand side captures expected marginal utility from keeping cash worth one unit of the security; the right-hand side captures expected marginal utility from buying the unit; for optimality, the two expected marginal utilities have to be the same. Formula (6) obtains by re-arrangement of the above equation. Under risk neutrality, and with  $\beta = 5/6$ ,  $p_{k,t} = 2.5$  for  $k \in \{\text{Tree, Bond}\}$

$$\begin{aligned}
&= \beta E\left[\frac{u'_i(c_{i,t+1})}{u'_i(c_{i,t})}d_{k,t+1}\right] \sum_{\tau=0}^{\infty} \beta^\tau \\
&= \frac{\beta}{1-\beta} E\left[\frac{u'_i(c_{i,t+1})}{u'_i(c_{i,t})}d_{k,t+1}\right],
\end{aligned}$$

which is the same as Eqn. (6). The task for the subjects was to trade off cash against securities. Cash is needed because it constituted experiment earnings if a period ended up to be terminal. Securities, in contrast, generated cash in future periods, for in case a current period was not terminal. It was easy for subjects to grasp the essence of the task. The simplicity allowed us to make instructions short. See Appendix for sample instructions.

There is one further difficulty which we have not mentioned: default. In the (finite or infinite horizon) Radner model, assets are simply promises; selling an asset – borrowing – entails a promise to buy the asset – repay – in the future. However, in the model, nothing enforces these promises: that they are kept in equilibrium is simply part of the definition of equilibrium. If nothing enforced these promises in the laboratory then participants could (and in our experience, would) simply make promises that they could not keep. One possibility for dealing with this problem is to impose penalties for default – failing to keep promises. In some sense that is what Radner equilibrium implicitly presumes: there are penalties for default and these penalties are so great that no one ever defaults. However imposing penalties is highly problematic in the laboratory for a number of reasons. What should the punishment be? The rules governing experimentation with human subjects prevent us from forcing subjects to pay from their own pockets, and excluding subjects from further participation in the experiment would raise a host of problems following such an exclusion – to say nothing of the fact that neither of these penalties might be enough to guarantee that default would not occur and to make it common knowledge that default would not occur. Moreover, this speaks only to *intentional default*, but what about *unintentional default* – *mistakes*? And what about plans that would have led to default in circumstances that might have occurred but did not? And what about the fact that the mechanisms for discouraging default might change behavior in other – unexpected – ways?

There is no simple solution to this problem because it is not a problem confined to the laboratory. Radner equilibrium effectively prohibits default but it is entirely silent about how this prohibition is to be enforced. As Kehoe and Levine (1993) and Geanakoplos and Zame (2007) (and others) have pointed out, mechanisms for dealing with default may eliminate default – but only at the cost of other distortions.

Our solution in the laboratory is to prohibit short-sales (negative holdings) of assets.

This creates a potential problem because the equilibrium analysis of Section 3 presumed that it was always possible for any agent to buy or sell an infinitesimal additional quantity of either asset, but if an agent's current holding of an asset were 0 he could not sell it and if his consumption and portfolio were both 0 he could not buy it. However, so long as agents do not bump up against the zero bound, the analysis of Section 3 remains correct; in the actual experimental data, the number of agents who bumped up against the zero bound was quite small. In our analysis, therefore, we shall simply take note of the prohibition of short sales but assume that the prohibition is never binding.

Because income and dividends, and hence, cash, fluctuated across periods, and cash was taken away as long as a period was not terminal, subjects had to constantly trade. As we shall see, trading volume was indeed uniformly high. In line with Crockett and Duffy (2010), we think that this kept serious pricing anomalies such as bubbles from emerging. Trading took place through an anonymous, electronic continuous open book system. The trading screen, part of software called Flex-E-Markets,<sup>10</sup> was intuitive, requiring little instruction. Rather, subjects quickly familiarized themselves with key aspects of trading in the open-book mechanism (bids, asks, cancellations, transaction determination protocol, etc.) through one mock replication of our economy during the instructional phase of the experiment. A snapshot of the trading screen is re-produced in Figure 1.

Shortsales were not allowed because they introduce the possibility of default. We already discussed the problems with default in the laboratory. Our barring shortsales explains why, contrary to Lucas' original model, the Bond is in positive net supply. This way, more risk tolerant subjects could merely reduce their holdings of Bonds rather than having to sell short (which was not permitted). Allowing for a second asset in positive supply only affects the equilibrium quantitatively, not qualitatively.<sup>11</sup>

All accounting and trading was done in U.S. dollars. Thus, subjects did not have to convert from imaginary experiment money to real-life currency.

We ran as many replications as possible within the time allotted to the experimental session. In order to avoid wealth effects on subject preferences, we paid for only a fixed number (say, 2) of the replications, randomly chosen after conclusion of the experiment. (If we ran less replications than this fixed number, we paid multiples of some or all of

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<sup>10</sup>Flex-E-Markets is documented at <http://www.flexemarkets.com/site>; the software is freely available to academics upon request.

<sup>11</sup>Because both assets are in positive supply, our economy is an example of a Lucas *orchard* economy (Martin, 2011).

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Table 3: Summary data, all experimental sessions.

Session	Place	Number of Replications	Number of Periods (Total within Session, Min. across Replications, Maximum)	Subject Count
1	Caltech*	4	(14, 1, 7)	16
2	Caltech	2	(13, 4, 9)	12
3	UCLA*	3	(12, 3, 6)	30
4	UCLA*	2	(14, 6, 8)	24
5	Caltech*	2	(12, 2, 10)	20
6	Utah*	2	(15, 6, 9)	24
(Overall)		15	(80, 1, 10)	

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the replications.)

## 4 Results

We conducted six experimental sessions, with the participant number ranging between 12 and 30. Three sessions were conducted at Caltech, two at UCLA, and one at the University of Utah. This generated 80 periods in total, spread over 15 replications. Table 3 provides specifics. Whenever the end of the experiment occurred during a replication, our novel termination protocol was applied: in the terminal period of these replications, participants knew for certain that it was the last period and hence, generated no trade. In the table, these sessions are starred. In other (unstarred) sessions, the last replication occurred sufficiently close to the end of the experiment that a new replication was not begun, so our termination protocol was not applied.

We first discuss volume, and then look at prices and choices.

**Volume.** Table 4 lists average trading volume per period (excluding terminal periods in which should be no trade). Consistent with theoretical predictions, trading volume in Periods 1 and 2 is significantly higher; it reflects trading needed for agents to move to their steady-state holdings. In the theory, subsequent trade takes place only to smooth consumption across odd and even periods. Volume in the Bond is significantly lower in Periods 1 and 2. This is an artefact of the few replications when the state in

Table 4: Trading volume.

Periods	Tree	Bond
	Trade Volume	Trade Volume
All		
<b>Mean</b>	<b>23</b>	<b>17</b>
St. Dev.	12	11
Min	3	2
Max	59	58
1 and 2		
<b>Mean</b>	<b>30</b>	<b>21</b>
St. Dev.	15	14
Min	5	4
Max	59	58
$\geq 3$		
<b>Mean</b>	<b>19</b>	<b>15</b>
St. Dev.	8	9
Min	3	2
Max	36	41

Period 1 was low. It deprived Type I participants of cash (Type I participants start with 10 Trees and no income). In principle, they should have been able to sell enough Trees to buy Bonds, but evidently they did not manage to complete all the necessary trades in the allotted time (four minutes). Across all periods, 23 Trees and 17 Bonds were traded on average. With an average supply of 210 securities of each type, this means that roughly 10% of available securities was turned over each period.<sup>12</sup> Overall, the sizeable volume is therefore consistent with theoretical predictions. To put this differently: we designed the experiment such that it would be in the best interest for subjects to trade every period, and subjects evidently did trade a lot.

**Cross-Sectional Price Differences.** Table 5 displays average period transaction prices as well as the period's state (High if the dividend of the Tree was \$1; Low if it was \$0). Consistent with the Lucas model, the Bond is priced *above* the Tree, with a

<sup>12</sup>Since trading lasted on average 210 seconds each period, one transaction occurred approximately every 5 seconds.

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Table 5: Period-average transaction prices and corresponding discount of the Tree price relative to the Bond price.

	Tree Price	Bond Price	Discount
Mean	2.75	3.25	0.50
St. Dev.	0.41	0.49	0.40
Min	1.86	2.29	-0.20
Max	3.70	4.32	1.79

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price differential of about \$0.50.

**Prices Over Time.** Figure 2 shows a plot of the evolution of (average) prices over time, arranged chronologically by experimental sessions (numbered as in Table 3); replications within a session are concatenated. The plot reveals that prices are volatile. In theory, prices should move only because of variability in economic fundamentals, which in this case amounts to changes in the dividend of the Tree. Prices should be high in High states, and low in Low states. In reality, a large fraction of price movements is unrelated to fundamentals; following LeRoy and Porter (1981) and Shiller (1981), we will refer to this as *excessive volatility*. Some price drift can be detected, but formal tests reported below will reveal that the drift is entirely due to the impact of states on prices, and the particular sampling of the states across the sessions.

Despite excessive volatility, evidence in favor of the Lucas model emerges. As Table 6 shows, prices in the high state are on average 0.24 (Tree) and 0.14 (Bond) above those in the low state. That is, prices do appear to move with fundamentals (dividends). The table does not display statistical information because (average) transaction prices are not i.i.d., so that we cannot rely on standard  $t$  tests to determine significance. We will provide formal statistical evidence later on.

Table 6 also shows that the discount on the Tree price relative to the Bond price is higher in periods when the state is Low than when it is High. This is inconsistent with the theory. Indeed, the prediction is quite the opposite: the discount should be pro-cyclical, and hence, higher in the High state.

**Cross-Sectional And Time Series Price Properties Together.** The theory predicts that the differential in prices between High and Low states should correlate positively with the difference between the Bond price and the Tree price, i.e., the discount of the Tree price relative to the Bond price. Correlation is to be taken across

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Table 6: Mean period-average transaction prices and corresponding discount of the Tree price relative to the Bond price, as a function of state.

State	Tree Price	Bond Price	Discount
High	2.91	3.34	0.43
Low	2.66	3.20	0.54
Difference	0.24	0.14	-0.11

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Table 7: Correlation across replications between the average discount on the Tree price relative to the Bond price and the average price differential of the Tree or Bond between High and Low states.

	Tree	Bond
Correlation	0.80	0.52
(St. Err.)	(0.40)	(0.40)

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economies, where economies are distinguished only by session cohort. Table 7 displays correlations of the average discount on the Tree price relative to the Bond price (regardless of state) and the average difference between prices of the Tree or of the Bond across states. Each observation corresponds to one replication, so there are 15 observations in total. Consistent with the theoretical prediction, the correlations are positive, though the estimate is insignificant for the Bond.

**Prices: Formal Statistics.** To enable formal statistical statements about the price differences across states, we ran a regression of period transaction price levels onto the state (=1 if high; 0 if low). To adjust for time series dependence evident in Figure 2, we added session dummies and a time trend (Period number). In addition, to gauge the effect of our session termination protocol, we added a dummy for periods when we announce that the session is about to come to a close, and hence, the period is either the penultimate or last one, depending on the draw of the die. Lastly, we add a dummy for even periods. Table 8 displays the results.

We confirm the positive effect of the state on price levels. Moving from a Low to a High state increases the price of the Tree by \$0.24, while the Bond price increases by \$0.11. The former is the same number as in Table 6; the latter is a bit lower. The

Table 8: OLS regression of period-average transaction price levels on several explanatory variables, including state dummy. (\* = significant at  $p = 0.05$ ; DW = Durbin-Watson statistic of time dependence of the error term.)

Explanatory Variables	Tree Price		Bond Price	
	Estim.	(95% Conf. Int.)	Estim.	(95% Conf. Int.)
Session Dummies:				
1	2.69*	(2.53, 2.84)	3.17*	(2.93, 3.41)
2	2.69*	(2.51, 2.87)	3.31*	(3.04, 3.59)
3	1.91*	(1.75, 2.08)	2.49*	(2.23, 2.74)
4	2.67*	(2.50, 2.84)	2.92*	(2.66, 3.18)
5	2.47*	(2.27, 2.67)	2.86*	(2.56, 3.17)
6	2.23*	(2.05, 2.40)	3.42*	(3.16, 3.69)
Period Number	0.06*	(0.03, 0.08)	0.06*	(0.01, 0.10)
State Dummy (High=1)	0.24*	(0.12, 0.35)	0.11	(-0.07, 0.29)
Initiate Termination	-0.07	(-0.28, 0.14)	-0.01	(-0.33, 0.31)
Dummy Even Periods	-0.00	(-0.11, 0.11)	-0.11	(-0.28, 0.06)
$R^2$		0.71		0.52
DW		1.05*		0.88*



Table 9: OLS regression of changes in period-average transaction prices. (\* = significant at  $p = 0.05$ .)

Explanatory Variables	Tree Price Change		Bond Price Change	
	Estim.	(95% Conf. Int.)	Estim.	(95% Conf. Int.)
Change in State Dummy (None=0; High-to-Low=-1, Low-to-High=+1)	0.19*	(0.08, 0.29)	0.10	(-0.03, 0.23)
$R^2$		0.18		0.04
Autocor. (s.e.=0.13)		0.18		-0.19

price increase is significant ( $p = 0.05$ ) for the Tree, but *not* for the Bond.

The coefficient to the termination dummy is insignificant, suggesting that our termination protocol is neutral, as predicted by the Lucas model. This constitutes comforting evidence that our experimental design was correct.

However, closer inspection of the properties of the error term revealed substantial dependence over time, despite our including dummies to mitigate time series effects. Table 8 shows Durbin-Watson (DW) test statistics with value that correspond to  $p < 0.001$ . Therefore, the results displayed in Table 8 must be treated with caution.

Further model specification analysis was performed, to ensure that the error term became properly behaved. This revealed that the best model involved first differencing price changes. All dummies could be deleted, and the highest  $R^2$  was obtained for a model that predicted price changes across periods as the result of only the change in the state. See Table 9.<sup>13</sup> For the Tree, the effect of a change in state from Low to High is significant ( $p < 0.05$ ) and substantial (\$0.19). The effect of a change in state on the Bond price is lower (\$0.10), though insignificant ( $p > 0.05$ ). Both confirm the theoretical prediction that prices should be determined by the state. The regression does not include an intercept; average price changes are insignificantly different from zero once the change in the state is accounted for. This implies that the apparent intra-session drift in the visual display of the price data (Figure 2) is entirely due to sampling of the states. The autocorrelations of the error terms are now acceptable (comfortably within two standard errors from zero).

The excessive volatility of prices is apparent from Table 9. Fundamentals (changes

<sup>13</sup>We deleted observations that straddled two replications. Hence, the results in Table 9 are solely based on intra-replication price behavior.

in the state) explain only 18% of the variability of the Tree prices ( $R^2 = 0.18$ ). This means that 82% of price variance is left unexplained, while the Lucas model predicts that zero variance should remain after taking into account the impact of the state.<sup>14</sup> The situation is even worse for the Bond: 96% of the variance of Bond price changes cannot be explained by changes in the state. It deserves emphasis that the unexplained variability is essentially noise; in particular, it is unrelated to the subject cohort, because session dummies were insignificant.

Overall, the regression in first differences shows that, consistent with the Lucas model, fundamental economic forces are behind price changes, significantly so for the Tree. But at the same time, prices are excessively volatile, with no distinct drift.

**Consumption Across States.** In the Lucas equilibrium, consumption choices are Pareto optimal. This means, in particular, that agents of both types should trade to holdings that generate high consumption in High states, and low consumption in Low states. Table 10 displays the average amount of cash (consumption) per type in High vs. Low states.<sup>15</sup> Consistent with the theoretical prediction, consumption is positively rank-correlated across Types. To gauge the significance of this finding, Table 10 also displays, in parentheses, the consumption (cash) levels that agents could have reached if they were not allowed to trade. These are the consumption levels under autarky. Note that consumption levels are *anti-correlated*. Through trading, the average Type I and Type II agents manage to move their consumptions from negatively to positively correlated, suggesting economically significant Pareto improvements, consistent with the Lucas model.

**Consumption Across Odd And Even Periods.** Another prediction is that subjects should be able to perfectly offset income differences across odd and even periods. Table 10 demonstrates that our subjects indeed managed to smooth consumption substantially; the outcomes are far more balanced than under autarky (numbers in parentheses; averaged across High and Low states, excluding Periods 1 and 2). Therefore, the experimental results suggest substantial Pareto improvements through trading.<sup>16</sup>

**Consumption Shares Across States and Across Odd and Even Periods.**

If one is willing to entertain the assumption that utilities of our subjects are homoth-

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<sup>14</sup>The fact that we related price changes to state changes using a linear model does not change this conclusion; there are only two states, so linearity obtains without loss of generality.

<sup>15</sup>To compute these averages, we ignored Periods 1 and 2, to allow subjects time to trade from their initial holdings to steady state positions.

<sup>16</sup>Autarky consumption of Type II subjects is not affected by states, because they are endowed with Bonds which always pay \$0.50 in dividends. In contrast, autarky consumption of Type I subjects depends on states. We used the sequence of realized states across all the sessions to compute their autarky consumption.

Table 10: Average consumption (end-of-period cash, in dollars) across states (High or Low Tree dividend) and across periods (Odd/Even), stratified by participant Type. Autarky numbers in parentheses. Last two rows:  $p$  levels of the contribution of State and Period to explaining variation of the consumption share of Type I (end-of-period cash holdings as a proportion of total cash available) in a two-way mixed-effects ANOVA (Analysis of Variance). For choices to be Pareto efficient, consumption shares should be independent of State and Period (provided the representative agents for the two participants Types are equal).

	States		Periods	
	High	Low	Odd	Even
Type I	14.93 (19.75)	7.64 (4.69)	7.69 (2.41)	13.91 (20.65)
Type II	15.07 (10.25)	12.36 (15.31)	14.72 (20)	11.74 (5)
ANOVA $p$ -value	0.09		0.27	
ANOVA Interaction $p$ -value	0.23			

etic, Pareto efficiency suggests a stronger prediction than positive (rank) correlation of consumption across states, or smoothing of income across Odd and Even periods. Under homothetic utilities, consumption shares should be independent across states and across periods. Table 10 displays the results of a formal test of equality of the consumption share of the average Type I subject across states and periods. The share of total consumption (total cash available) that the average Type I subject chose at the end of each period was computed and a two-way analysis of variance (ANOVA) was applied, with state (High/Low) and period (Odd/Even) as potential factors determining variability in this consumption share, allowing for interaction between state and period. A mixed-effects approach was used, to accommodate differences in consumption shares across replications due to differences in drawing of the state in the first period and in subject cohort.

Table 10 shows that neither the state nor the nature of the period (nor their interaction) are significant factors ( $p > 0.05$ ) in explaining the variability of the consumption share of the average Type I subject across periods. As such, the apparent violations of the prediction of equal consumption shares across states/periods implied by the average consumption levels reported in Table 10 are solely due to sampling error.

The finding is rather striking, because the assumption of homothetic preferences is questionable. Yet, our empirical results suggest that the assumption can be maintained as far as the choices of the average subject of Type I (and by implication, of Type II)

Table 11: End-of-period asset holdings, type I subjects. Averages across all replications and subjects (of Type I). Initial allocation in parentheses, for reference.

Period	1	2	3	4	5	6	7	8	9
\$ Income	0	15	0	15	0	15	0	15	0
Asset:									
Tree (10)	6.67	7.00	5.67	6.33	5.75	6.75	5.92	6.67	6.92
Bond (0)	0	1.08	0.33	1.25	0.50	1.60	0.92	2.58	2.25
Total (10)	6.67	8.08	6.00	7.58	6.25	8.35	6.84	9.25	9.17

are concerned.

**Price Hedging.** The above results suggest that our subjects (on average) managed to move towards the Pareto-optimal equilibrium consumption patterns of the Lucas model. However, contrary to model prediction, they did not resort to price hedging as a means to ensure those patterns. Table 11 lists average asset holdings across periods for Type I subjects (who received income in Even periods). They were net sellers of assets in periods of income shortfall (see “Total” row). But unlike in the theory, they *decreased* Tree holdings in low-income periods and *increased* them in high-income periods. Only in period 9 is there some evidence of price hedging: Type I subjects on average *bought* Trees when they were income-poor (Period 9’s holding of Trees is higher than Period 8’s).

**Subject-Level Differences.** There are, however, significant individual differences in portfolio choices. Table 12 illustrates how three subjects of Type I end up holding almost opposing portfolios of Trees and Bonds. Subject 7 increased his holdings of Trees over time. Significantly, this subject bought Trees even in periods with income shortfall (odd periods), effectively implementing the price hedging strategy of the theory. Subject 5 is almost a mirror image of subject 7, though s/he did not resort to price hedging. Subject 3 diversified across Trees and Bonds but did not hedge price risk either because Tree holdings decreased in odd periods.

The subject-level differences reported in Table 12 are no exception. The contrast between choices at the individual level and at the Type level is sharp. The theory “works” at the Type level, but not at the individual level. This contrast suggests that one has to be careful extrapolating to phenomena at the market level (e.g., prices) from observing individuals singly. If we had taking any of the three subjects as “typical,” and had predicted cross-sectional and temporal behavior of prices on the basis of their

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Table 12: End-Of-Period Asset Holdings Of Three Type I Subjects. Initial allocations: 10 Trees, 0 Bonds. Data from one replication in the first Caltech session.

Subject	1	2	3	4	5	6
Trees:						
3	4	4	3	4	3	4
5	1	1	0	1	1	3
7	7	10	13	15	19	20
Bonds:						
3	3	5	3	5	3	4
5	8	15	14	15	16	17
7	2	3	0	4	0	4

---

choices, the fit would have been poor. The situation is reminiscent of the cross-sectional variation in choices in static asset pricing experiments. There too, prices at the market level can be “right” (satisfy, e.g., CAPM) even if individual choices are at odds with the theory; see Bossaerts et al. (2007a).

## 5 The Expected and the Anomalous

With respect to the predictions of the Lucas model, our experiments generate findings that are expected – smoothing of individual consumption across states and time, correlation of individual consumption with aggregate consumption, even to the extent that consumption shares are independent of state and period – and findings that seem anomalous – excessive volatility of prices and absence of price hedging by subjects. Co-existence of excessive volatility with the absence of price hedging might be surprising: it would seem that excessive volatility would signal especially clearly to subjects that they ought to hedge against price risk. However, this need not be so; the particular kind of excessive volatility that we see in the experimental data might well lead subjects to conclude that there is no need to hedge against price risk.

To see why this might be so, recall first that the predictions of the Lucas model and indeed the very definition of Radner equilibrium depend on the assumption that agents have perfect foresight and in particular that the beliefs of subjects about the dividend process and the price process are *exactly correct*. Because these processes

must be learned, it would be too much to expect that beliefs be exactly correct but perhaps not too much to expect that beliefs be *approximately* correct. Optimization against exactly correct beliefs leads exactly to the Radner equilibrium predicted by the Lucas model, and it would seem that optimization against approximately correct beliefs should lead something that approximates the Radner equilibrium predicted by the Lucas model. However, this is not so: because the price process is *endogenous*, beliefs about the price process can be approximately correct even though the actual price process is very far from the price process predicted by the Lucas model. Precisely the same point has been made by Adam et al. (2012), who used it to explain excessive volatility in historical data, just as we use it to make sense of excessive volatility in our experimental data.<sup>17</sup>

On the basis of our experimental data, it seems quite plausible that agents expected prices to follow a martingale – as would be predicted by the (naive version of the) Efficient Markets Hypothesis – and *not* to co-move with economic fundamentals – as would be predicted by the Lucas model. This belief is wrong, but it is not readily falsifiable on the basis of the limited number of observations available to subjects. Indeed, the belief that Bond prices do not follow a martingale would not be falsifiable even after 80 observations – an order of magnitude more observations than were available to subjects. The belief that prices follow a martingale is thus a credible working hypothesis.

A thought experiment may help to understand the consequences of these incorrect beliefs. Imagine that in every period agents always believe that past prices are the best predictions of future prices, *independently of economic fundamentals*; that, given these beliefs, agents correctly solve their current optimal investment-consumption problem as a function of prices; that agents then send demand schedules to the market; and that the market generates prices to that demand and supply are equal in that period. Of course, beliefs are wrong and will be revealed to be wrong next period, so we are considering in this thought experiment only a kind of temporary equilibrium, but one in which beliefs, although incorrect, are disciplined by observation. How would prices in this temporary equilibrium evolve over time? Simulations suggest that prices would evolve very much as in the experiment: they do co-move with dividends, but very noisily – hence they are excessively volatile.

Figure 3 displays the evolution of prices and states in a typical simulation of this temporary equilibrium. There are two types of agents, endowed as in the experiment. Both are represented by an agent with logarithmic utility. Agent beliefs (that prices

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<sup>17</sup>Most analyses of the Lucas model that have addressed beliefs that are only approximately correct have considered only beliefs about the *exogenous* dividend process; see Hassan and Mertens (2010) for instance.

revert to the levels of the previous period) are affected every period by an additive gaussian disturbance with mean zero and standard deviation \$0.40. Agents start out believing that the Tree will be priced at \$2.5 and the Bond at \$3. This produces price evolutions very much in line with those in the experiment. At the same time, agents do not hedge price risk (they don't perceive any and accommodate income shortfalls solely by selling Bonds *and* Trees). Still, their choices do move substantially towards Pareto optimality: the consumption share of the Type I agent fluctuates only between 39% and 44%, little affected by state and period (Odd/Even).

This thought experiment demonstrates starkly that the price predictions of the Lucas model are fragile to small mistakes in beliefs about the price process. This comes as a surprise because the price predictions of the Lucas model are robust to small mistakes in beliefs about the dividend process (Hassan and Mertens, 2010). The difference is that the price process is endogenous – so that mistakes can create positive feedback – while the dividend process is endogenous – so that mistakes are damped out. As mentioned before, Adam et al. (2012) derived an analogous result, and showed that it provides a good rationale for excessive volatility of historical real-world stock market prices.

## 6 The Data Viewed Through A Traditional Lens

An interesting exercise is to run traditional Generalized Method of Moments (GMM) tests on our laboratory data. We will pretend that the data are like historical data from the field – limited to time series of asset returns and aggregate consumption – and test whether the first-order conditions (“stochastic Euler equations”) are satisfied for a representative agent. We effectively ignore that we know more. We have information on individual choices, endowments and consumption. We know the true payout probabilities and we know the true state of the world (whether the dividend on the Tree is high or low). Etc. But we shall ignore all that momentarily.

We assume that the representative agent has power utility and will estimate the coefficient of risk aversion ( $\gamma$ ;  $\gamma = 1$  corresponds to log utility) and the discount factor ( $\beta$ ) while testing whether the Euler equations hold, assuming that the representative agent “consumes” the aggregate cash each period.

It should be emphasized that our assuming power utility is without loss of generality. Since we have only two states, in equilibrium only the marginal rate of substitution in moving from a Low to a High state needs to be estimated. We have used  $x$  to denote this marginal rate of substitution. The marginal rates of substitutions of the reverse

state transition is just the reciprocal ( $1/x$ ); the marginal rates of substitutions for the remaining transitions equal 1. As such, only one parameter needs to be estimated (besides the impatience factor  $\beta$ ). One could as well assume power utility, which effectively means that  $x$  is modeled as a power of the ratio of consumption in the High and the Low state. This is what our GMM estimation will accomplish.

The Euler equations are:

$$E\left[\beta \left(\frac{c_{t+1}^A}{c_t^A}\right)^{-\gamma} \frac{d_k + p_{k,t+1}}{p_{k,t}} - 1 \mid I_t\right] = 0,$$

where  $c_t^A$  and  $c_{t+1}^A$  denote aggregate (per capita) total cash in periods  $t$  and  $t + 1$ , respectively,  $k \in \{B, T\}$ , and  $I_t$  is any information that agents in the economy (participants in our experiments) had at the end of period  $t$ . As is standard in GMM tests of these Euler equations, we choose variables in the agents' information set, called "instruments" and denoted  $z_t$ . Each instrument generates a set of two unconditional moment conditions (one for each of the assets,  $B$  and  $T$ ), by applying the law of iterated expectations:

$$\begin{aligned} E[E\left[\left(\beta \left(\frac{c_{t+1}^A}{c_t^A}\right)^{-\gamma} \frac{d_k + p_{k,t+1}}{p_{k,t}} - 1\right) z_t \mid I_t\right]] &= E\left[\left(\beta \left(\frac{c_{t+1}^A}{c_t^A}\right)^{-\gamma} \frac{d_k + p_{k,t+1}}{p_{k,t}} - 1\right) z_t\right] \\ &= 0. \end{aligned}$$

Each choice of instruments leads to a different test.

Our *first test* is based on a traditional instrument choice, going back to Hansen and Singleton (1983). Specifically, we choose as instruments (i) the constant 1, (ii) lagged consumption growth, and (iii)-(iv) lagged returns on the Tree  $T$  and Bond  $B$ . Thus, we have 4 instruments, and since each instrument generates 2 moment conditions, we have 8 moment conditions in total. Only two parameters ( $\beta, \gamma$ ) need to be estimated, so we have 6 over-identifying restrictions. The idea behind GMM is to find values of the parameters that minimizes a quadratic form in the moment conditions. With a suitable weighting matrix, the resulting minimum is  $\chi^2$  distributed, with degrees of freedom equal to the number of over-identifying restrictions.<sup>18</sup> The necessary time series, of consumption growth and asset returns, were constructed by concatenating periods across all replications and all sessions, leaving out observations that would straddle two different replications, as we did for Table 9.

The top panel of Table 13 displays the results of the first test. Three observations stand out.

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<sup>18</sup>We implemented GMM using Matlab routines provided by Michael Cliff.



Table 13: GMM Estimation And Testing Results For Three Different Sets Of Instruments.

Instruments	$\beta$ ( $p$ value for $\beta = 5/6$ )	$\gamma$ ( $p$ value for $\gamma = 0$ )	$\chi^2$ test ( $p$ value)
constant 1, lagged consumption growth, lagged asset returns	0.86 (0.003)	-0.01 (0.917)	7.124 (0.310)
constant 1, lagged consumption growth	0.86 (0.029)	-0.18 (0.162)	0.731 (0.694)
high state dummy, low state dummy, lagged consumption growth	0.86 (0.002)	0.16 (0.001)	14.349 (0.006)

1. The model is not rejected ( $p = 0.310$ ), while it should be rejected because prices display a large component of unexplained variation (excessive volatility).
2. The discount factor is significantly above the theoretical one ( $5/6$ ); this may suggest that participants over-estimated the probability of continuation of a replication, or equivalently, under-estimated the chance that a replication would terminate.
3. The coefficient of risk aversion ( $\gamma$ ) is insignificantly different from zero, suggesting participants were risk neutral, despite ample evidence reported before that (i) the price of the Tree is lower than that of the Bond, (ii) participants smoothed consumption (end-of-period cash holdings) substantially, both across states (high and low) and across odd and even periods; both observations support the hypothesis that participants were risk averse.

It is fair to state that the GMM results are misleading. The GMM test (of over-identifying moment restrictions) gives one the false impression that pricing is entirely in accordance with the Lucas model, while the estimated risk aversion is inconsistent with subject choices. This situation is reminiscent of that in Asparouhova (2006). In the context of a competitive market for loans under adverse selection, Asparouhova (2006) used standard structural estimation using the Rothschild-Stiglitz equilibrium pricing model, and failed to reject its predictions when the model was factually false, and moreover, obtained parameter estimates that were significantly different from the truth.

The estimate of the risk aversion coefficient, at  $-0.01$ , suggest risk neutrality, or even risk seeking. Closer inspection of the data suggests why. While average returns on the securities are positive (not surprisingly; this reflects the significant discount rate), and contrary to the theoretical predictions, the average return on the Tree, at 12.8%, is below that of the Bond (15.9%). This perverse ranking of expected returns is consistent with risk seeking attitudes, however, consistent with the estimated parameters.

The  $\chi^2$  GMM test of over-identifying restrictions is suspect, though. Two instruments, the lagged return on the Tree and Bond, are actually “weak” instruments, in the sense that they are uncorrelated, even independent, over time, both with themselves and with consumption growth. (Details can be obtained from the authors upon request.) As such, the corresponding moment conditions reduce to those with a constant as instrument. That is, these moment conditions do not provide additional restrictions beyond the ones imposed by the moment conditions constructed with the constant as instrument. Effectively, the number of degrees of freedom in the  $\chi^2$  test is not 6, but only 2.

To determine the impact of these weak instruments, we ran a *second test*, re-estimating the model with only the constant and lagged consumption as instruments. The second panel of Table 13 displays the results. As expected, the estimation results ( $\beta$ ,  $\gamma$ ) hardly change. The test of over-identifying restrictions still fails to reject, however. Consequently, the effect of the weak instruments is nil; they could as well be deleted, but adding them does not change the conclusions.

The GMM test with traditional instruments does not exploit all restrictions of the model. In particular, expected returns are predicted to be different across states (high/low Tree dividend), but consumption growth can only capture the *change* in the state, and not the value of the state. In historical data from the field, consumption growth is readily observable (though there is a debate whether the right consumption series is being used), but not the state itself. Here, we are in control of the state, and hence, can use it as an instrument. Our choosing the state as instruments is inspired by the finding that the discount of the Tree price relative to the Bond price is counter-cyclical, in contrast to the theoretical prediction. (As we shall discuss below, the equity premium – which is the analog in terms of returns – is pro-cyclical, in violation of the theory.) GMM may be able to pick up this perverse result and thereby the reject the model.

Consequently, in our *third test*, we replaced the constant instrument with two dummy variables, one that tracked the high state, and the other one tracking the low state. We kept the remaining instrument, the consumption growth. In total, this gives

three instruments and as such generated six moment conditions. With two parameters to estimate, we are left with four degrees of freedom.

The results are presented in the bottom panel of Table 13. We observe the following.

1. The model is now rejected, at the 1% level.
2. The discount factor,  $\beta$ , continues to be a bit too high, though.
3. Risk aversion is now significant (at 1% level).

Further inspection sheds light on why the GMM test (which is based on moment restrictions on asset returns) rejects. At estimated parameters, most moment conditions fit tightly. However, the moment condition involving the Tree return and using the dummy variable for the High state does not fit well. And indeed, the average return on the Tree in the High state is lower (at 12.8%) than that of the Bond (at 15.9%). Thus, the equity premium in the High state is *negative*, and this can only be fit with a negative risk aversion coefficient. In contrast, in the Low state, the ranking of returns is consistent with risk aversion: 17.8% for the Tree and 16.1% for the Bond. GMM estimates a positive risk aversion coefficient, allows it to fit well the moment conditions in the Low state for both assets, as well as the moment condition for the Bond in the High state (which is lower than in the Low state, consistent with the theoretical prediction). Notice also that the equity premium is pro-cyclical, contrary to the theory (but in line with the counter-cyclical nature of the discount of the Tree price against the Bond price).

## 7 Conclusion

Over the last thirty years, the Lucas model has become *the* core theoretical model through which scholars of macroeconomics and finance view the real world, advise investments in general and retirement savings in particular, prescribe economic and financial policy and induce confidence in financial markets. Despite this, little is known about the true relevance of the Lucas model. The recent turmoil in financial markets and the effects it had on the real economy has severely shaken the belief that the Lucas model has anything to say about financial markets. Calls are being made to return to pre-Lucas macroeconomics, based on reduced-form Keynesian thinking. This paper was prompted by the belief that proper understanding of whether the Lucas model (and the Neoclassical thinking underlying it) is or is not appropriate would be enormously advanced if we could see whether the model did or did not work in the laboratory.

Of course, it is a long way from the laboratory to the real world, but it should be kept in mind that no one has ever seen convincing evidence of the Lucas model “at work” – just as no one had seen convincing evidence of another key model of finance (the Capital Asset Pricing Model or CAPM) at work until the authors (and their collaborators) generated this evidence in the laboratory (Asparouhova et al., 2003; Bossaerts and Plott, 2004; Bossaerts et al., 2007a). The research provides absolutely crucial – albeit modest – evidence concerning the scientific validity of the core asset pricing model underlying formal macroeconomic and financial thinking.

Specifically, despite their complexity, our experimental financial markets exhibited many features that are characteristic of the Lucas model, such as the co-existence of a significant equity premium and (albeit reduced) co-movement of prices and economic fundamentals. Consistent with the model, the co-movement increased with the magnitude of the equity premium. And subjects managed to smooth consumption over time and across states, to the extent that we could not reject that the consumption share of the average Type I subject was independent of state and period. As such, consumption choices displayed a major feature of Pareto optimality – even if this feature required utilities to be homothetic, something we had no control over.

Prices were excessively volatile though (not unlike in the real world, incidentally). And we did not observe price hedging, perhaps because subjects believed that the best predictor for future prices were past prices (reminiscent of a naive version of EMH?). Still, such beliefs were not irrational: within the time frame of a single replication, there was insufficient evidence to the contrary, because the excess volatility made it hard to determine to what extent prices really reacted to fundamentals.

Overall, we view our experiments as a success for the Lucas model. Note that this model is only a reduced-form version of a general equilibrium. It assumes that markets somehow manage to reach Pareto optimal allocations, but is silent about how to get there. In our experiment, markets were incomplete, which makes attainment of Pareto optimality all the more challenging – markets had to be dynamically complete, and sophisticated trading strategies were required. Our design was in part mandated by experimental considerations: with incomplete markets, subjects had to trade every period. Crockett and Duffy (2010) have demonstrated that subjects need a serious reason to trade in all periods, otherwise pricing anomalies (bubbles) emerge. But we would also argue that realistic markets are generically incomplete, and as such, our experiments provide an ecologically relevant test of the Lucas model.

Real-world financial markets are thought to be excessively volatile, and policy makers have long been worried about this. On the policy side, our experimental results

raise the issue whether excessive volatility is of concern if welfare is the goal. The possibility that excessive volatility may not matter much has so far escaped the attention of empiricists and theorists, perhaps because evaluation of Pareto optimality cannot be performed on field data – because the stochastic Euler equations with which field prices are evaluated *assume* Pareto efficiency.

Overall, we have a rather paradoxical situation, where consumption choices at the aggregate level (average per type) appear Pareto optimal, and hence, are consistent with the Lucas model, yet prices are “wrong” (i.e., not supported by those choices). This situation is rather unsettling, because the Lucas model is almost exclusively used to explain prices, while its prediction regarding choices – Pareto optimality – is taken as a maintained assumption. Our data suggest an explanation for the paradox. The Lucas model requires agents to make correct forecasts of future prices. In the model, this boils down to determining the mapping from states to prices. In reality, agents make mistakes, and these mistakes un-do the tight link between states and prices. As such, additional risk emerges that is absent in the Lucas model. Our results suggests that further exploration of a modeling approach along the lines of Adam et al. (2012) would be fruitful; there, agents are allowed to make (small) mistakes in forecasting future prices.

Since price expectations are crucial for the Lucas model to obtain, it may be worth asking whether a change in the asset mix would both facilitate welfare enhancements and improve model fit. Specifically, one-period riskfree notes could replace our (perpetual) Bond. This way, agents do not have to worry about forecasting the re-sale price of one of the securities. From an experimental point of view, substituting one-period riskfree notes for a perpetual bond is not straightforward (because the notes need to be re-issued every period). But the idea is worth pursuing in future research, and may eventually shed light on an important issue: what is the optimal maturity of riskfree securities?

Our experimental results also illustrate that it is dangerous to extrapolate from the individual to the market. As in our static experiments (Bossaerts et al., 2007b), we find substantial heterogeneity in choices across subjects; most individual choices have little or no explanatory power for market prices, or even for choices averaged across subjects of the same type (same endowments). Overall, the system (market) behaves as in the theory (modulo excessive price volatility), but the theory is hardly reflected in individual choices. As such, we would caution against developing asset pricing theories where the system is a mirror image of (one of) its parts. For instance, it is doubtful that prices in financial markets would reflect, say, prospect theoretic preferences (Barberis

et al., 2001), merely because many humans exhibit such preferences (leaving aside the problem that these preferences do not easily aggregate). The “laws” of the (financial) system are different from those of its parts.

Our experiments allowed us to perform an exercise that sheds light on the merits of traditional methodology for the analysis of historical field data. The idea was pioneered in Asparouhova (2006). Here, we studied our experimental data through the lens of the familiar Generalized Method of Moments (GMM) tests of the “stochastic Euler equations” that restrict pricing to be in line with aggregate consumption growth. The inference turned out to be misleading. Not only did we fail to reject the null (which we should in view of excessive volatility), our parameter estimates were at odds with the other evidence (price levels; choices) we could observe in the experimental data – but to which researchers in the field have no access. Asparouhova (2006) draws the same conclusion in a different setting.

Another step in our experimental analysis could be to reduce the termination probability, thus generating longer time series. This way, one could study to what extent markets eventually converge to the Lucas equilibrium. The reader may wonder why we have not done so already. One reason is experimental. We wanted to make sure that subjects understood that any period, including the first one, could be terminal. To be credible, we needed to generate a few cases where termination occurred early on (one of the replications in the first session terminated after one period, and we did not fail to mention this during the instruction phase of the subsequent sessions). This required a high termination probability. The second reason is based on personal opinion. We do not believe that the real world is stationary. Parameters change before full convergence to the Lucas equilibrium. As such, it is irrelevant to ask what happens in Lucas economies of long duration. Despite the short horizon, we find it remarkable that our experimental markets manage to generate results that are very much in line with general equilibrium theory (suitably modified to explain the noise prices). Nevertheless, eventual convergence to the Lucas equilibrium is an interesting theoretical possibility, and here again, laboratory experiments could be informative.

# Appendix: Instructions (Type I Only)

Web Address: [filagora.caltech.edu/fm/](http://filagora.caltech.edu/fm/)

User name:

Password:

## INSTRUCTIONS

### 1. Situation

One session of the experiment consists of a number of replications of the same situation, referred to as *periods*.

You will be allocated *securities* that you can carry through all periods. You will also be given *cash*, but cash will not carry over from one period to another.

Every period, markets open and you will be free to *trade* your securities. You buy securities with cash and you get cash if you sell securities.

Cash is not carried over across periods, but there will be two sources of fresh cash in a new period. First, the securities you are holding at the end of the previous period may pay *dividends*. These dividends become cash for the subsequent period. Second, before the start of specific periods, you may be given *income*. This income becomes cash for the period. It will be known beforehand in which periods you receive income.

Each period lasts 5 minutes. The total number of periods is not known beforehand. Instead, at the end of a period, we determine whether the experiment continues, as follows. We throw a twelve-sided die. If the outcome is 7 or 8, we terminate the session. Otherwise we continue and advance to the next period. Notice: the termination chance is time-invariant; it does not depend on how long the experiment has been going.

**Your experiment earnings are determined by the cash you are holding at the end of the period in which the session ends.**

So, if you end a period without cash, and we terminate the session at that point, you will not earn any money for the session. This does not mean, however, that you should ensure that you always end with only cash and no securities. For in that case, if we continue the experiment, you will not receive dividends, and hence, you start the subsequent period without cash (and no securities) unless this is a period when you receive income.

We will run as many sessions as can be fit in the allotted time of two hours for the experiment. If the last session we run has not been terminated before the scheduled end of the experiment we will terminate the session and you will earn the cash you are holding at that point.

You will be paid the earnings of two randomly chosen sessions. If we manage to run only one session during the allotted time for the experiment, you will be paid double the earnings for that session.

During the experiment, accounting is done in real dollars.

## 2. Data

There will be two types of securities, called **tree** and **bond**. One unit of the tree pays a random *dividend of zero or one dollar*, with equal chance; past dividends have no influence on this chance. (The actual draw is obtained using the standard pseudo-random number generator in the program “matlab”). One unit of the bond *always pays fifty cents*. You will receive the dividends on your holdings of trees and bonds in cash **before** a new period starts. As such, you will receive dividends on your initial allocation of trees and bonds before the first period starts.

You will start this session with **10 trees** and **0 bonds**. Others may start with different initial allocations.

In addition, you will receive income every alternate period. In **odd** periods (1,3...) you will receive nothing, and in **even** periods (2,4,...) you will receive 15 dollars. This income is added to your cash at the beginning of a new period. Others may have a different income flow.

Because cash is taken away at the end of a period when the session does not terminate, the dividend payments you receive, together with your income, are the sole sources of cash for a new period.

## 3. Examples (for illustration only)

Tables 1 and 2 give two sample examples of outcomes in a session. It is assumed that the session ends after the 6<sup>th</sup> period. Table 1 shows the asset holdings, dividend and cash each period if the states are as per row 2 and the individual sticks to the initial allocation throughout. The final take-away cash/earning is 25 dollars as the session terminated after 6<sup>th</sup> period. It would have been \$0 if it had terminated in period 5.

Table 2 shows the case where the individual trades as follows:

- In period 1, to an allocation of 5 trees and 5 bonds,
- And subsequently, selling to acquire more cash if dividends and income are deemed too low,
- Or buying more assets when dividends and income are high.

Since there is a 1/6 chance that the session ends in the period when a security is bought, its expected value equals  $(\frac{5/6}{1-5/6})$  times the expected dividend (which is equal for both the tree and the bond), or  $(5) * (0.5) = 2.50$ . Trade is assumed to take place at 2.50. Note, however, that the actual trading prices may be different, and that they may even change over time, depending on, e.g., the dividend on the tree. The final take-away cash in this case is \$15.00. It would have been \$13.00 if the session had terminated in period 5.



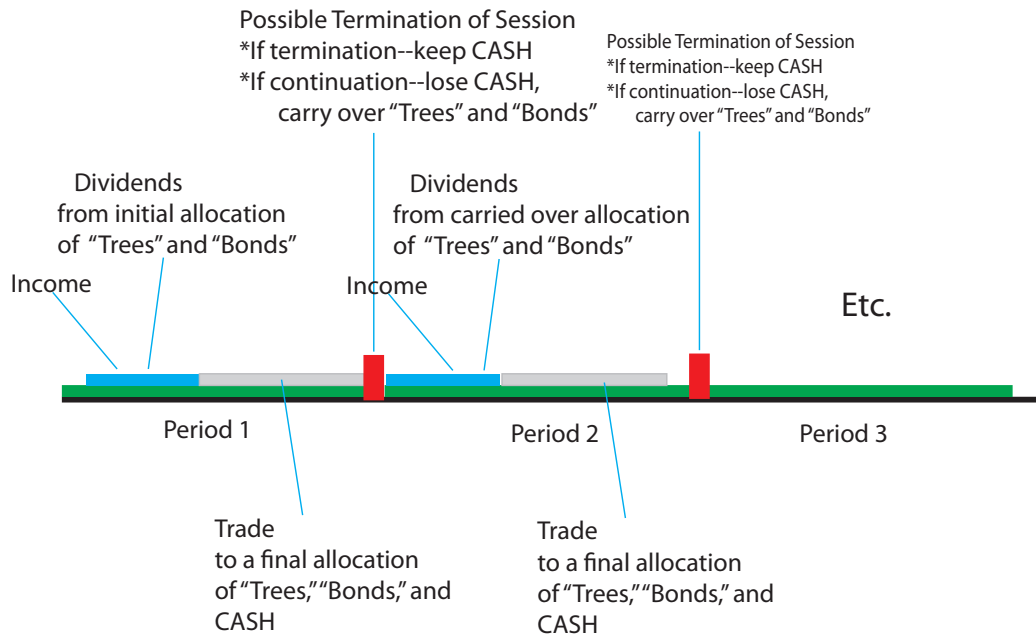
Table 1.

PERIOD	1	2	3	4	5	6
State	H	L	L	H	L	H
<b>Initial Holdings</b>						
Tree	10	10	10	10	10	10
Bond	0	0	0	0	0	0
<b>Dividends</b>						
Tree	$\$1 \cdot 10 = 10$	$\$0 \cdot 10 = 0$	$\$0 \cdot 10 = 0$	$\$1 \cdot 10 = 10$	$\$0 \cdot 10 = 0$	$\$1 \cdot 10 = 10$
Bond	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 0 = 0$
<b>Income</b>	0	15	0	15	0	15
<b>Initial Cash</b>	\$10 (=10+0+0)	\$15 (=0+0+15)	\$0 (=0+0+0)	\$25 (=10+0+15)	\$0 (=0+0+0)	\$25 (=10+0+15)
<b>Trade</b>						
Tree	0	0	0	0	0	0
Bond	0	0	0	0	0	0
<b>Cash Change</b>	\$0	\$0	\$0	\$0	\$0	\$0
<b>Final Holdings</b>						
Tree	10	10	10	10	10	10
Bond	0	0	0	0	0	0
<b>CASH</b>	<b>\$ 10.00</b>	<b>\$ 15.00</b>	<b>\$ 0.00</b>	<b>\$ 25.00</b>	<b>\$ 0.00</b>	<b>\$ 25.00</b>

Table 2.

PERIOD	1	2	3	4	5	6
State	H	L	L	H	L	H
<b>Initial Holdings</b>						
Tree	10	5	6	4	5	3
Bond	0	5	6	4	6	4
<b>Dividends</b>						
Tree	$\$1 \cdot 10 = 10$	$\$0 \cdot 5 = 0$	$\$0 \cdot 6 = 0$	$\$1 \cdot 4 = 4$	$\$0 \cdot 5 = 0$	$\$1 \cdot 3 = 3$
Bond	$\$0.5 \cdot 0 = 0$	$\$0.5 \cdot 5 = 2.5$	$\$0.5 \cdot 6 = 3$	$\$0.5 \cdot 4 = 2$	$\$0.5 \cdot 6 = 3$	$\$0.5 \cdot 4 = 2$
<b>Income</b>	\$0	\$15	\$0	\$15	\$0	\$15
<b>Initial Cash</b>	\$10 (=10+0+0)	\$17.5 (=0+2.5+15)	\$3 (=0+3+0)	\$21 (=4+2+15)	\$3 (=0+3+0)	\$20 (=3+2+15)
<b>Trade</b>						
Tree	-5	+1	-2	+1	-2	+1
Bond	+5	+1	-2	+2	-2	+1
<b>Cash Change</b>	\$0	-\$5	+\$10	-\$7.5	+\$10	-\$5
<b>Final Holdings</b>						
Tree	5	6	4	5	3	4
Bond	5	6	4	6	4	5
<b>CASH</b>	<b>\$ 10.00</b>	<b>\$ 12.50</b>	<b>\$ 13.00</b>	<b>\$ 13.50</b>	<b>\$ 13.00</b>	<b>\$ 15.00</b>

## Appendix: Time Line Plot To Complement Instructions



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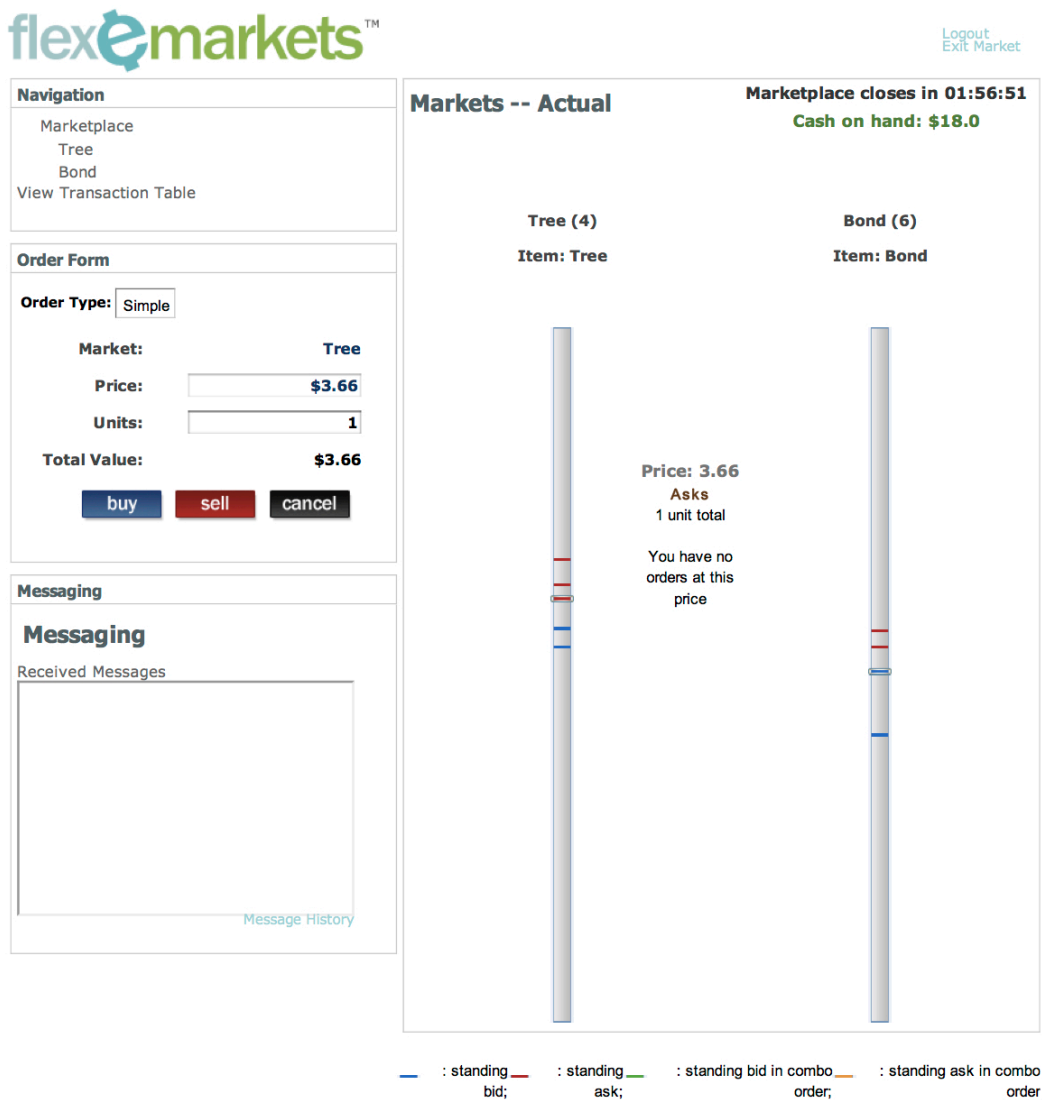
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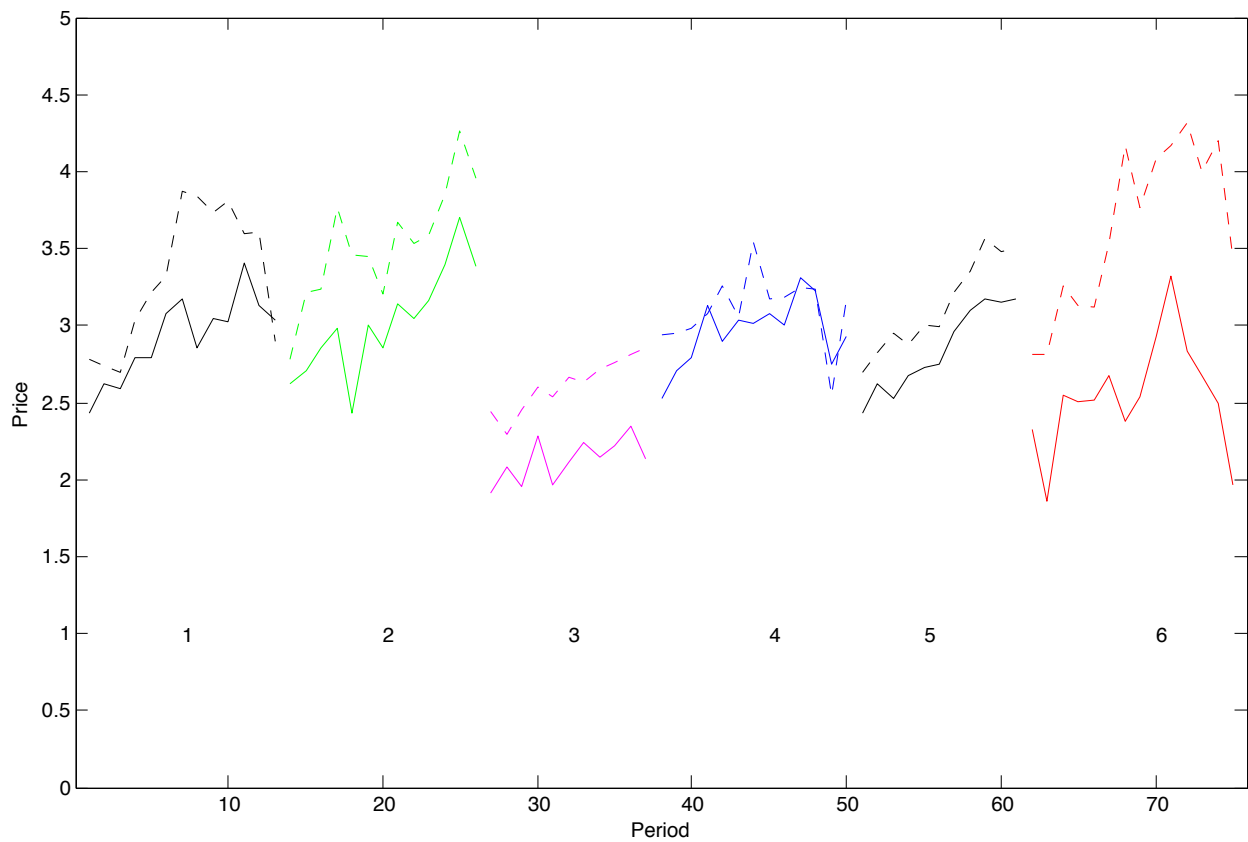
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Figure 1: Snapshot of the trading interface. Two bars graphically represent the book of the market in Trees (left) and in Bonds (right). Red tags indicate standing asks; blue tags indicate standing bids. Detailed information about standing orders is provided by clicking along either of the bars (here, the Tree bar is clicked, at a price level of \$3.66). At the same time, this populates the order form to the left, through which subjects could submit or cancel orders. Asset holdings are indicated next to the name of the market, and cash balances are given in the top right corner of the interface. The remaining functionality in the trading interface is useful but non-essential.



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Figure 2: Time series of Tree (solid line) and Bond (dashed line) transaction prices; averages per period. Session numbers underneath line segments refer to Table 3.





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Figure 3: Time series of Tree and Bond prices in a temporary equilibrium where agents expect prices to revert back to last period's levels, plus mean-zero gaussian noise with \$0.40 standard deviation. Also shown is the evolution of the state (High = 1; Low = 0).

